ADVANTAGES AND LIMITATIONS OF DIMENSIONAL ANALYSIS

Advantages

1. Dimensional analysis is a useful tool in the analysis and correlation of experimental data in the planning of experiments and in the formulation of empirical correlation describing a particular phenomenon.

2. The presentation of data in non-dimensional groups allows the application of empirical correlation to a wide range physical conditions. Further, the non-dimensional data is suitable for use in any consistent system of units.

3. The important variables involved in a physical phenomenon are systematically organised into dimensionless groups which are less numerous from the original variables. Few experiments then needed to be conducted to cover a wide range of parameters and that results in a considerably saving in time, money and efforts.

\[ h = f(\varepsilon, \zeta, k, C_p, V, d) \]

Through dimensional analysis , we would obtained

\[ \frac{hd}{k} = C \left( \frac{V \rho}{\mu}, \frac{C_p \mu}{k} \right) \]
ADVANTAGES AND LIMITATIONS OF DIMENSIONAL ANALYSIS

\[ h = f(\varepsilon, \rho, k, C_p, V, d) \]

If the effect of each original variables upon the convective coefficient is to be investigated individually, then the test run will have to be made with say 5 different cylinder diameters and conducting experiments with each of the cylinder with 5 fluids of different thermal conductivities further vary fluids density 5 times and so on.

This implies that \(5^6 = 15625\) tests will have to be conducted which is rather cumbersome 5 times conducting.

With the aid of dimensional analysis, the original variables have been results into a dimensional groups obviously. The same information can now be obtained through \(5^2 = 25\) experiments.

\[
\frac{hd}{k} = C\left(\frac{V}{\mu}, \frac{C_p \mu}{k}\right)
\]
Limitations:

1. Information from previous experiments is necessary to known the different variables which are likely to affect the phenomenon being analysis.

1. No information is given about the internal mechanism of the physical phenomenon
Reynolds number (Re)

- Ratio of the inertial force to the viscous force.
- It is inductive of the relative importance of inertial and viscous effect in a fluid motion.
- At low Reynolds number the viscous effect dominates the fluid motion is laminar.
- At high Reynolds number the inertial effects lead to turbulent flow.

\[
Re = \frac{\rho u D}{\mu}
\]
SIGNIFICANCE OF DIMENSIONLESS GROUPS

Grashof number (Gr)

Â Gr indicates the **relative strength of the buoyant to viscous forces**.

Â From its mathematical formulation,

\[ Gr = \frac{l^3 g\beta \rho^2 \Delta T}{\mu^2} \]

\[ = (l^3 g\beta \rho \Delta T) \frac{\rho}{\mu^2} \]

\[ = (l^3 \rho g \Delta T) \times \frac{\rho V^2 l^2}{(\mu Vl)^2} \]

\[ = \text{buoyant force} \times \frac{\text{inertia force}}{(\text{viscous force})^2} \]

Â Grashof number represents the **ratio of the product of buoyant and inertia force to the square of the viscous force**.

Â Grashof number has a role in free convection similar to that played by **Raynolds number in forced convection**.
SIGNIFICANCE OF DIMENSIONLESS GROUPS

Prandtl number (Pr)

Â It is indicative of the relative ability of the fluid to diffuse momentum and inertial energy by molecular mechanisms.

From its mathematical formulation,

\[ \text{Pr} = \frac{\mu C_p}{k} = \frac{\rho \gamma C_p}{k} = \frac{\gamma}{\kappa} \]

Â Recalling that the parameter \( \frac{k}{\rho C_p} \) is thermal diffusivity of the fluid

\[ \text{Pr} = \frac{\gamma}{\kappa} \]

Â Pr is the ratio of the kinematic viscosity to thermal diffusivity of the fluid.

Â The kinematic viscosity indicates the momentum transport by molecular friction and thermal diffusivity represents the heat energy transport through conduction.
SIGNIFICANCE OF DIMENSIONLESS GROUPS

Å The Prandtl number is connecting link between the velocity field and the temperature field and its value strongly influences relative growth of velocity and thermal boundary layers.

Mathematically,

\[ \frac{\delta}{\delta_t} \approx (Pr)^n \]

Å Where \( \delta \) and \( \delta_t \) are thickness of velocity and thermal boundary layers respectively and \( n \) is a positive exponent.

For oils \( \delta_t \ll \delta \)

For gases \( \delta_t \approx \delta \)

and for metals \( \delta_t \gg \delta \)
SIGNIFICANCE OF DIMENSIONLESS GROUPS

Nusselt number (Nu)

Å Nusselt establishes the relation between convective film coefficient $h$, thermal conductivity of the fluid $k$ and a significant length parameter $l$ of the physical system.

$$ Nu = \left( \frac{hl}{k} \right) $$

Å Energy balance at the surface of a heated plate stipulates that energy transport by conduction must equal the convective heat transfer into the fluid flowing past the plate. Thus

$$ Q = -kA \left( \frac{\partial t}{\partial y} \right)_{y=0} = hA(t_s - t_{\infty}) $$

$$ h = \frac{-k \left( \frac{\partial t}{\partial y} \right)_{y=0}}{(t_s - t_{\infty})} $$

$$ \frac{hl}{k} = \left( \frac{-\partial t/\partial y}{(t_s - t_{\infty})/l} \right)_{y=0} $$
SIGNIFICANCE OF DIMENSIONLESS GROUPS

\[
\frac{hl}{k} = \left( -\frac{\partial t}{\partial y} \right)_{y=0}
\]

Å Nu may be interpreted as the ratio of temperature gradient at the surface to an overall reference temperature gradient.

Å The nusselt number is a convenient measure of the convective heat transfer coefficient.

\[ h \propto k \]

\[ h \propto 1/l \]
SIGNIFICANCE OF DIMENSIONLESS GROUPS

Stanton number (St)

It is the ratio of heat transfer coefficient to the flow of heat per unit temperature rise due to the velocity of the fluid.

\[ St = \frac{h}{\rho V C_p} = \frac{h l / k}{\rho V l \times \mu / C_p} = \frac{Nu}{Re \times Pr} \]

\[ St = \frac{Nu}{Re \times Pr} \times \frac{Prandtl \text{ number}}{Reynolds \text{ number}} \]

It should be noted that Stanton number can be used only in correlating forced convection data.

Through dimensional analysis, we have obtained the following possible forms for correlation of convection data:

(a) Forced convection  \( Nu = f_1(Re, Pr) \)

or

\( St = f_2(Re, Pr) \)

(b) Free convection  \( Nu = f_3(Gr, Pr) \)